Logarithms & Indices

Leaving Certificate Higher Level Maths

All Logs and Indices Formulae are on page 21 of the Formulae & Tables Book

<u>Point to Note:</u> **Logs** and **Indices** are closely linked. Raising a base to a power and finding the log to the base are **inverse operations**!!

$2^3 = 8 \leftrightarrow \text{Log}_2 8 = 3$

2 is the base!

3 is the power!

8 is the number!

Formula in Table Book for converting logs to indices or vice versa

$y = a^n \leftrightarrow \text{Log}_a y = n$

Examine the below table and consider the functions f (x) = 2^x and g(x) – $\log_2 x$

x (input values)	$f(x)$ (output values)	x (input values)	$g(x)$ (output values)
-1	$2^{-1} = \frac{1}{2}$	$\frac{1}{2}$	$\log_2 \frac{1}{2} = -1$
0	$2^0 = 1$	1	$\log_2 1 = 0$
1	$2^1 = 2$	2	$\log_2 2 = 1$
2	$2^2 = 4$	4	$\log_2 4 = 2$
3	$2^3 = 8$	8	$\log_2 8 = 3$

<u>Laws of Indices (Page 21)</u>

$a^p a^q = a^{p+q}$

$\frac{a^p}{a^q} = a^{p-q}$

$(a^p)^q = a^{pq}$

$a^0 = 1$

$a^{-p} = \frac{1}{a^p}$

$a^{\frac{1}{q}} = \sqrt[q]{a}$

$a^{\frac{p}{q}} = \sqrt[q]{a^p} = \left(\sqrt[q]{a}\right)^p$

$(ab)^p = a^p b^p$

$\left(\frac{a}{b}\right)^p = \frac{a^p}{b^p}$

Some of the Laws are demonstrated in the below table:

Law 1 (product rule)	$3^2 \times 3^4 = 3^{2+4} = 3^6$
Law 2 (quotient rule)	$\frac{7^6}{7^3} = 7^{6-3} = 7^3$
Law 3 (power to a power)	$(4^2)^3 = 4^{2\times3} = 4^6$
Law 4 (zero exponent)	$6^0 = 1$
Law 5 (n^{th} root)	$27^{\frac{1}{3}} = \sqrt[3]{27} = 3$
Law 6 (fractional exponent)	$4^{\frac{3}{2}} = (\sqrt[2]{4})^3 = 8$
Law 7 (negative exponent)	$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$
Law 8	$(2 \times 3)^6 = 2^6 \times 3^6$
Law 9	$\left(\frac{4}{3}\right)^5 = \frac{4^5}{3^5}$

Point to Note when solving Index Equations:

$a^x = a^y \rightarrow x = y$

As long as the same base is on both sides we can equate indices.

Laws of Logs (Page 21)

$$\log_a(xy) = \log_a x + \log_a y$$

$$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$$

$$\log_a(x^q) = q\log_a x$$

$$\log_a 1 = 0$$

$$\log_a\left(\frac{1}{x}\right) = -\log_a x$$

$$a^x = y \Leftrightarrow \log_a y = x$$

$$\log_a(a^x) = x$$

$$a^{\log_a x} = x$$

$$\log_b x = \frac{\log_a x}{\log_a b}$$

SOME LEAVING CERTIFICATE EXAM QUESTIONS

Given $\log_a 2 = p$ and $\log_a 3 = q$, where $a > 0$, write each of the following in terms of p and q:

(i) $\log_a \frac{8}{3}$

$$p = \log_a 2 \ , \qquad q = \log_a 3$$

$$\log_a \frac{8}{3} = \log_a 8 - \log_a 3$$

$$= \log_a (2)^3 - \log_a 3$$

$$= 3\log_a 2 - \log_a 3$$

$$= 3p - q$$

$\log_a \frac{9a^2}{16}$.

$$\log_a \frac{9a^2}{16} = \log_a (3a)^2 - \log_a (2)^4$$

$$= 2\log_a 3 + 2\log_a a - 4\log_a 2$$

$$= 2q + 2(1) - 4p$$

$$= 2q + 2 - 4p$$

Given that $p = \log_c x$, express $\log_c \sqrt{x} + \log_c (cx)$ in terms of p.

We know that

$$\log_c \sqrt{x} = \log_c x^{\frac{1}{2}} = \frac{1}{2}\log_c x = \frac{1}{2}p$$

using the power law for logarithms.
Also,

$$\log_c (cx) = \log_c c + \log_c x = \log_c c + p$$

using the product rule for logarithms.
But $\log_c c = 1$ since $c^1 = c$. Therefore

$$\log_c \sqrt{x} + \log_c (cx) = \frac{1}{2}p + 1 + p = \frac{3p}{2} + 1.$$

Scientists can estimate the age of certain ancient items by measuring the proportion of carbon−14, relative to the total carbon content in the item. The formula used is $Q = e^{-\frac{0\cdot 693t}{5730}}$, where Q is the proportion of carbon−14 remaining and t is the age, in years, of the item.

(a) An item is 2000 years old. Use the formula to find the proportion of carbon−14 in the item.

$$Q = e^{-\frac{0\cdot 693t}{5730}} = e^{-\frac{0\cdot 693\times 2000}{5730}} = 0\cdot 7851$$

(b) The proportion of carbon−14 in an item found at Lough Boora, County Offaly, was 0·3402. Estimate, correct to two significant figures, the age of the item.

$$Q = e^{-\frac{0\cdot 693t}{5730}} = 0\cdot 3402$$

$$\Rightarrow -\frac{0\cdot 693t}{5730} = \ln 0\cdot 3402$$

$$\Rightarrow t = -\frac{5730\times \ln 0\cdot 3402}{0\cdot 693} \approx 8915 \approx 8900 \text{ years}$$

If $\log_{\sqrt{2}} x + \log_{\sqrt{2}}(x+4) = 4$ x > 0, Find X

Question 2 (c) (i)

$$\log_{\sqrt{2}} x + \log_{\sqrt{2}}(x+4) = 4$$

$$\log_{\sqrt{2}} x(x+4) = 4$$

$$x^2 + 4x = (\sqrt{2})^4 = (2^{\frac{1}{2}})^4 = 2^2 = 4$$

$$x^2 + 4x - 4 = 0$$

$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(-4)}}{2(1)} = \frac{-4 \pm \sqrt{16+16}}{2}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-4 \pm \sqrt{32}}{2} = \frac{-4 \pm 4\sqrt{2}}{2} = -2 \pm 2\sqrt{2}$$

$$x > 0 \Rightarrow x = 2\sqrt{2} - 2$$